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TEACHING MULTIPLICATION

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A VISUAL APPROACH

At Right Angles
A Resource for School Mathematics

INTRODUCTION

TEACHING MULTIPLICATION

Here are some questions which arise while teaching Multiplication: Should children memorise the multiplication tables? What is an easy and convenient way of modeling multiplication? Is it enough if one only teaches the procedure of multiplication? Perhaps answers to these questions can be found if we reflect on the importance we give to construction of knowledge. If we see that children must understand how facts are derived, how procedures are derived and how concepts can be visualized, then our approach will be dictated by that understanding.

Keywords: *Multiplication, manipulatives, pattern, cycle, symmetry, commutative, Cartesian product*

We start with two 'warm up activities' before introducing multiplication (Activity 1 and Activity 2).

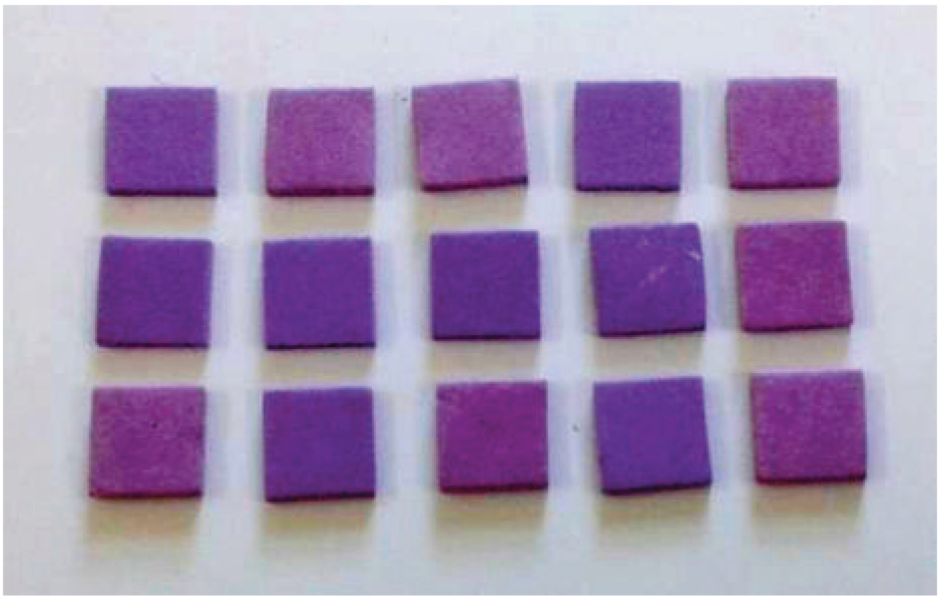
ACTIVITY *ONE*

Making equal groups and internalising multiplication contexts

Materials required: Square pieces, straws and rubber bands, coloured buttons. Peg board and pegs or a graph board and seeds

One group of children can work with straws and make bundles of straws with the same number of straws in each bundle. Another group can arrange square pieces in rows with the same number of pieces in each row. Yet another group can line up seeds on a graph board or square ruled sheets. (Seeds can also be placed in paper plates or bowls.) By rotation, all groups should work with different materials. Different children learn in different ways. We need to expose them to multiple ways of looking at things. Also, working with different materials and different arrangements will help children become familiar with different contexts in which multiplication arises. Further, it is important that children of this age group are exposed to tactile learning. This will aid in visualizing problems and strengthen their conceptual understanding. Doing these activities will also help children who learn through a kinesthetic approach.

The purpose of this activity is to focus on re-arranging objects into equal groups and distinguishing between the two numbers (the number of groups made, and the number in each group) arising from the situation. It is not necessary at this point to talk of the total number. Questions will centre around the following: 'How many groups?', 'How many in each group?'



ACTIVITY *TWO*

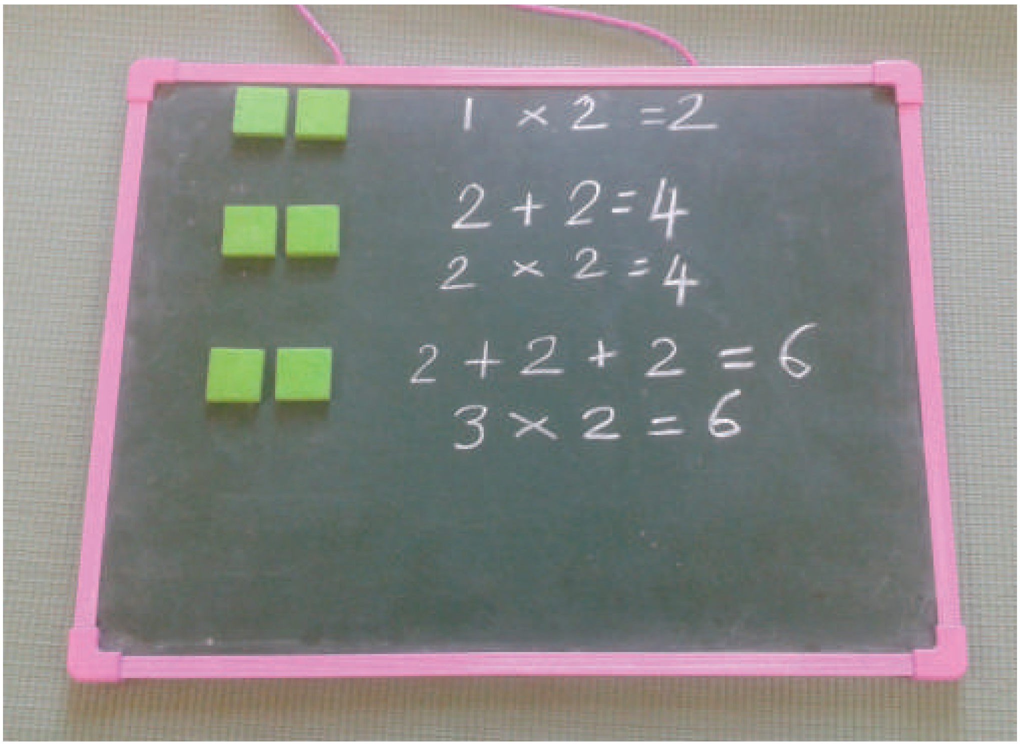
Skip counting in steps of 2, 5, 4 and 10

Materials required: String of beads, number line and number chart

With or without the aid of a number line, children can do skip counting using skips of 2, 5, 4 and 10. They could also try skip counting with other numbers if they are at ease with them. They can do forward counting as well as backward counting.

Here the questions will centre around the following: 'In what steps are we counting?' Say 2. 'How many steps of 2 did we count to reach 10?' Answer: 5 repetitions of 2 have brought us to 10.

It is fun to do this as a hopping activity on a number line drawn on the ground. Children can explore whether they can reach 12 by hopping in steps of 2 or 3, or steps of any other number.



ACTIVITY ***THREE***

Introduction to multiplication table 2 through repeated addition

Materials required: Seeds or square pieces

While introducing any multiplication table it is important to construct the table gradually in front of the children, articulating each step clearly.

Arrange 2 squares in a row and say: "This is 1 group of 2 squares." (One two is two, this is written as $1 \times 2 = 2$) Now place 2 squares under them, saying: "This is 2 groups of 2 squares" (two twos are four, this is written as $2 \times 2 = 4$). Now build the third row of 2 more squares (three twos are six, $3 \times 2 = 6$) and so on till ten twos are twenty, $10 \times 2 = 20$.

I prefer to teach multiplication tables as $1 \times 2 = 2$, $2 \times 2 = 4$, $3 \times 2 = 6$, $4 \times 2 = 8$, etc. (changing the first number and keeping the second number constant).It is the group number which increases each time while the group size remains constant. This corresponds to the way we speak about a multiplicative situation: 3 rows of 10 chairs, 4 classes of 20 students, five 2 kg packets of salt, etc.

However, if one prefers to teach the tables as $2 \times 1 = 2$, $2 \times 2 = 4$, $2 \times 3 = 6$, etc., then while arranging the squares in successive rows, one will have to say: 'two occurring once is 2, $2 \times 1 = 2$', 'two repeated twice is four, $2 \times 2 = 4$', 'two repeated thrice is six, $2 \times 3 = 6$', and so on.

Whichever approach one takes, one needs to proceed gradually, stating the number that is repeated and the number of times it is repeated.

Also, let children record their activities as drawings (as shown in the picture for Activity Two). It is important that they record the result both as a repeated addition and as a multiplication fact, in both forms till they internalize the relationship between repeated addition and multiplication.

Usage of 'into': For some reason, while reciting multiplication tables, the usage of the word 'into' has crept into our language ("2 into 4 equals 8"), but this is not appropriate. In fact, when one asks,"How many times does 2 go into 4?" it actually means *division* (4 divided by 2), and the answer is 2. We need to change this practice and read multiplication facts as "3 times 2 equals 6", "4 times 8 equals 32", etc.

Multiplication tables for 5, 4 and 3 (I prefer to teach the tables for 2 and 5 first) can be introduced in a similar manner. It is good to pause at this point and spend time consolidating these facts before we go on to further multiplication tables.

ACTIVITY **FOUR**

Patterns in multiplication tables of 2, 3, 4 and 5 as an aid in committing the tables to memory

Materials required: Multiplication Tables chart with bold numbers, number chart (1 to 100)

Discuss with the children the patterns seen in the multiplication table of 5. They can first look at the numeral in the units place and observe that 5 and 0 repeat in a cycle of 2. They will also notice that in the tens place, each number appears twice.

Next they can work on the pattern in the multiplication table of 2. They will see that numerals 2, 4, 6, 8, 0 repeat with a cycle of 5. But in the tens place, the pattern does not establish itself unless they build the table further. This is a good point to show an extended multiplication table (which we normally do not attempt).

Now they can work on the pattern in the multiplication table of 4. They will see that the numerals 4, 8, 2, 6, 0 repeat in the units place with a cycle of 5. What about the pattern in the tens place? Do we need to extend the table to notice a pattern? Is there any relationship between the sequence of digits in the units place of 4 table and the sequence of digits in the units place of 2 table?

Finally they look for patterns in the multiplication table of 3. The patterns can be found more easily if we group the digits of the units place in groups of three and place them in rows under one another:

3 6 9

2 5 8

1 4 7

Children will see that the digits of the first, second and third columns decrease by 1 each time.

Should Multiplication Tables Be Memorised?

First: Children should have plenty of exposure to the concept of multiplication and internalize it.

Second: Children should be able to build or construct any multiplication table with understanding.

Third: Usage of aural memory or visual memory in learning and memorising the multiplication tables of 2 to 10 is very useful in mental arithmetic and saves a lot of time.

Multiplication tables have also been set to tunes and are available in the market and on the internet as songs. It will help children who are musically inclined.

Many teachers either skip or rush through the first two steps in a cursory way and get children to memorise tables. This will not lead to an understanding of the concept and makes the child helpless whenever his memory fails. The capacity to build a table is enabling and empowering to the child.

Also, there is no need for panic if some children take more time to memorise. We want children to think in mathematical ways and not merely learn by rote. It is therefore advisable that we give a lot of attention to the proper understanding of this concept.

ACTIVITY *FIVE*

Constructing tables from 6 to 10

Materials required: Broom sticks or cardboard strips or plastic tongue cleaners

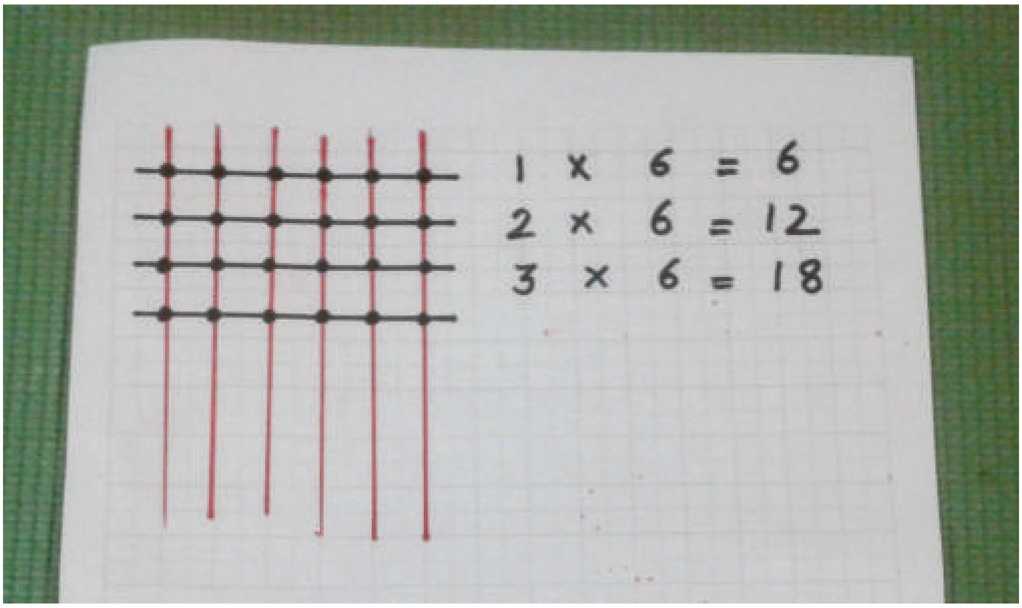
Multiplication facts for 6, 7, 8 and 9 can be taught using any of the following methods.

- Repeated addition, using seeds or buttons
- Arranging square pieces in array form (rows and columns)
- Counting the joints of intersecting lines

The third approach has the advantage of being less cumbersome than the first two methods when one is constructing a multiplication table for a larger number. It is also easier for children to make rough sketches of it in their notebooks.

Arrange 6 strips parallel to one another vertically. Lay one strip horizontally across them and point out the joints where they intersect and say $1 \times 6 = 6$. Lay one more strip horizontally across the vertical lines, point out the joints where they intersect and say $2 \times 6 = 12$. Lay one more strip horizontally across the vertical lines, point out the joints where they intersect and say $3 \times 6 = 18$, and so on.

Once children have understood the process by which they have created the multiplication table for 6, they will be able to do the same for 7, 8 and 9 on their own and work out the multiplication facts.



ACTIVITY *SIX*

Noting the patterns in multiplication tables of 6, 7, 8, 9 and 10 as an aid in committing the tables to memory

Materials required: Multiplication Tables chart with bold numbers, Number chart (1 to 100)

The pattern for table 10 is obvious.

Discuss with children the patterns they notice in the multiplication table of 9. It has many patterns and there is a lot that children will be able to discover on their own if the teacher poses some leading questions.

Finger pattern showing $9 \times 4 = 36$

Finger pattern showing $9 \times 5 = 45$

They can first look at the numeral in the units place and see that it goes down from 9 to 0. At the same time, the tens place increases from 1 to 9. The digits of the number always add up to 9. The units digits have a cycle of 10 before they repeat. The table can be demonstrated using the fingers of both hands in a simple fashion by progressively raising the first finger, followed by the second, etc, and reading tens from the left side of the raised finger and units from the right side of the raised finger, as shown in the figure.

They can now look for patterns in the multiplication table of 8. They will see that 8, 6, 4, 2, 0 repeat in the units place with a cycle of 5. But in the tens place, the pattern does not establish itself unless they extend the table further. Is there a relationship between the sequence of the digits in the units place of the 4 table and the sequence of digits in the units place of the 8 table?

Now they can work on the pattern in the multiplication table of 6. They will see that numerals 6, 2, 8, 4, 0 repeat in the units place with a cycle of 5. What about the pattern in the tens place? Will we need to extend the table to notice a pattern?

Finally they can look for patterns in the multiplication table of 7. The patterns can be easily found if we group the digits in the units place in threes and place them in rows, one below the other (like we did in the case of multiplication by 3):

7 4 1

8 5 2

9 6 3

The digits of the first column, second column and third column are seen to increase by 1 at each stage.

ACTIVITY *SEVEN*

Creating visual patterns using multiples of 2, 3, 4, 5, 6, 7, 8, 9

Materials required: Square grid paper, 8 sheets per child

Ask the children to write the numbers from 1 to 100 with a pencil in a 10 by 10 square grid.

Let them colour the multiples of 2 in their grids and note the pattern that emerges.

Let them write 1 to 100 again on another 10 by 10 square grid and this time colour all the multiples of 3. This creates a diagonal pattern.

They can repeat this exercise for other numbers 4 to 9 on different square grids.

Discuss the patterns that emerge.

ACTIVITY *EIGHT*

Discovering commutativity, associativity and distributive property of multiplication

Square grid paper, cardboard strips

COMMUTATIVITY

While we want children to discover these three properties of multiplication, we can avoid mentioning the names to young children and demonstrate only the property.

Let children make 5 groups of 3 seeds. Ask them what number this gives. Record the answer 5 x 3 = 15.

Let them now show 3 groups of 5 seeds. Ask them what number this gives. Record the answer 3 x 5 =15.

Ask them: "Is 3 groups of 5 each the same as 5 groups of 3 each?" What is common? It is the answer which is common.

Let children colour a row of 6 squares. Let them make 3 such rows. They can now record what they have coloured.

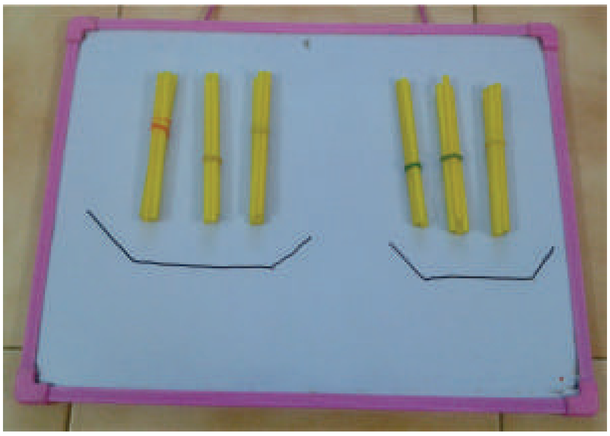
3 rows of 6 squares equals 18, i.e., $3 \times 6 = 18$.

Now ask them to turn the drawing through a right angle to make it vertical. Ask them to describe the number of rows that they see now.

They see 6 rows of 3 squares. So $6 \times 3 = 18$.

Now point out that 3×6 gives the same result as 6×3.

ASSOCIATIVITY



Ask children to bundle 4 straws together using a rubber band. Let them make 6 such bundles. Place them equally in two plates (i.e., 3 bundles in each plate). Now let us count the total number of straws.

There are 2 plates, 3 bundles in each plate, and 4 straws in each bundle.

The total number of straws can be calculated as the number of bundles times number of straws in each bundle, i.e., $(2 \times 3) \times 4$, or as the number of plates times the number of straws in one plate, i.e., $2 \times (3 \times 4)$. So: $(2 \times 3) \times 4 = 2 \times (3 \times 4)$.

DISTRIBUTIVITY

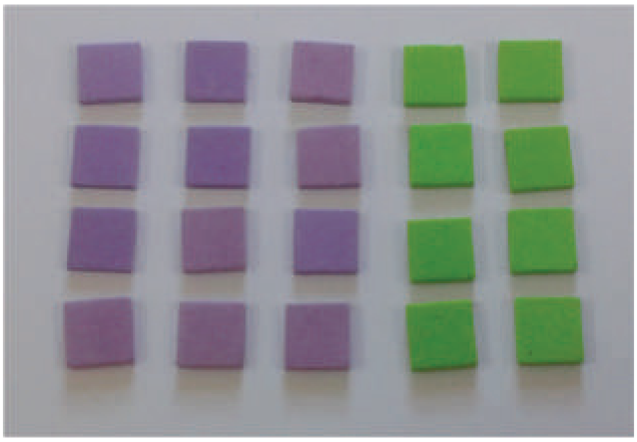
Let children colour the squares as shown. Let them state the multiplication fact for the purple squares (4 rows of 3 squares each, $4 \times 3 = 12$) and green squares (4 rows of 2 squares each, $4 \times 2 = 8$), separately.

Next, let them state the multiplication fact for the whole region: 4 rows of 5 squares each, $4 \times 5 = 20$.

Hence:

$(4 \times 3) + (4 \times 2) = 4 \times (3 + 2) = 4 \times 5 = 20$.

Several examples of each type need to be shown using various contexts and numbers for the three laws to be understood.



ACTIVITY **NINE**

Modelling word problems and writing stories for given multiplication facts.

Materials required: Square grid paper, plain paper, dot paper, seeds

Use all the different multiplication contexts

Equal groups: 4 bowls, 5 apples in each bowl. How many apples?

Rate: Every child needs 2 pencils. How many pencils for 24 children?

Arrays: 4 plants in each row; 3 rows. How many plants?

Scale factor: A boy has 4 books; his brother has 3 times as many. How many books does the brother have?

Cartesian product: A boy has 3 T-shirts (red, yellow, white) and 2 shorts (black, blue). What are the different ways in which he can pair them?

Many children have difficulties in interpreting word problems. This difficulty continues to persist in higher classes when they encounter word problems in linear equations or applications of percentages. Teaching modelling techniques for word problems is neglected both by text books and many teachers. Problems should be introduced in a contextual situation while we teach, and we also need to expose children to different modelling techniques.

1. A gardener plants 9 plants in each bed. There are 4 beds in the garden. How many plants?

   Let children in the initial stages model it using seeds, or use a dot paper to depict the seeds and beds.

2. A kitchen wall is covered with tiles. If there are 8 tiles in each row and the mason needs to make 7 such rows, how many tiles does he need?

   Children can work with square pieces or use square grid paper to model this.

3. Trees are planted at 5 metres distance from the start to the end of a street. If 8 trees are planted on the street, how long is the street?

   Children can use a number line to depict the situation and work out the answer.

For a scale factor they can depict it as a graph.

For a Cartesian product problem they can make a tree diagram or a network.

Writing Stories for multiplication facts

Ask children to write a story for a multiplication fact like $6 \times 5 = 30$. It will reveal their understanding or bring out misconceptions. I have always found this exercise very revealing; it gave me a chance to remedy my teaching. The contexts they choose will give us feedback on the kind of examples that we have used.

ACTIVITY *TEN* | Multiplication by 1 and 0

Sticks or straws

How does one explain to a child that $n \times 1 = n$ and $n \times 0 = 0$?

Providing a convincing explanation is not easy. If one gives repetition as an explanation, then one will be forced to say 2 occurring twice becomes four (2 times 2 = 4) and two occurring once is two (1 times 2 = 2). How about zero times 2?

One way is to use a flow technique in the reverse direction, by using sticks and counting the joints.

We start with 5×2, shown using sticks, and progressively remove one stick at a time, counting the joints each time: 5×2 (10 joints), 4×2 (8 joints), 3×2 (6 joints), 2×2 (4 joints), 1×2 (2 joints), 0×2 (0 joints). It is important that we provide consistent explanations which follow patterns and are logical.

CONCEPTUAL UNDERSTANDING AND PROCEDURAL UNDERSTANDING

Often teachers face a dilemma with regard to concepts and procedures. They are not sure whether they need to focus on one or the other. There are some (definitely a large number in India!) who think that procedures are more important, as they help in solving problems.Many techniques and shortcuts are taught. Some feel that in this day of calculators and computers, procedures are taken care of by gadgets, so they need to give importance only to conceptual understanding. However, procedures are a result of our historical research of methods, and if the logic behind the procedure is gone into by the teachers, along with the students, it addresses both the understanding of concepts and an appreciation of procedures. So we do not need to put procedures and concepts in an either-or situation.

Procedures are compatible with teaching of concepts. Students need to understand the conceptual system of multiplication in which procedures are fully integrated. Deriving procedures and finding generalizations can be rewarding at all levels and build the mathematical muscle of the brain.

ACTIVITY *ELEVEN*

Multiplication of a double digit number by a single digit number

Materials required: Place value Kit.

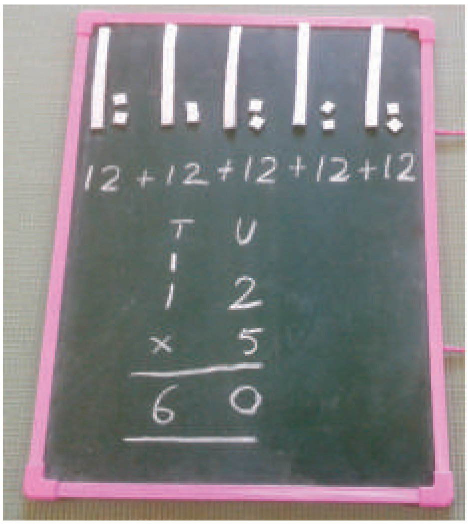
Introduction to multiplication of a double digit number by a single digit number is best done through place value material. One needs to constantly emphasize the place value aspect in all operations, as it determines the procedural knowledge needed for solving a problem. For example, in computing 32 x 8,when we multiply the digit in the units place (2) with 8 and write 1 over the number (3) in the tens place, that 1 stands for '1 ten'. In the next stage, when we multiply 3 by 8, we are actually multiplying 3 tens (30) by 8. The meaning of all this comes through only when we use place value materials and emphasize the place values coming into operation in each step.

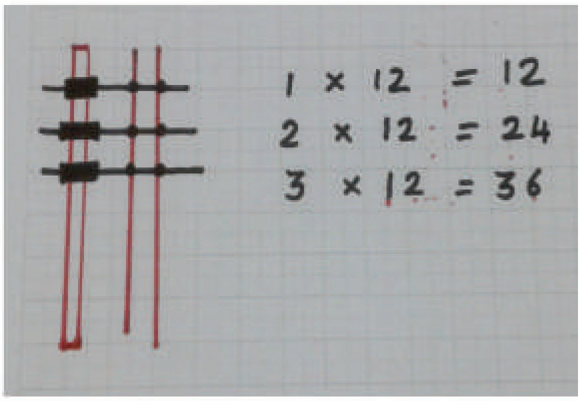
Initially, use examples which do not need exchange of units to tens:12 $\times$ 4 or 13 $\times$ 3, etc, using place value materials.

In the second stage, demonstrate 12 $\times$ 5 again with place value materials. Relate it to the procedure by showing the in-between steps for better understanding.

By focusing on the place value show that $12 \times 5 = (10 + 2) \times 5 = 10 \times 5 + 2 \times 5$.

It can also be shown by using thicker strips to indicate tens and thinner strips for units and counting the corresponding joints as tens and units, as shown in the picture.





Extension: Multiplication of a three digit number by a single digit number can be demonstrated using hundreds, tens, units material and should again be taught without any exchanges initially, followed by exchanges in tens place and later exchanges in hundreds place.

It can also be shown through expanded form: $324 \times 7 = (300 \times 7) + (20 \times 7) + (4 \times 7)$.

ACTIVITY *TWELVE* | Multiplication by 10 and multiples of 10

Multiplication by 10 comes easily to children.

Multiplication by multiples of ten (20, 30, 40) involves usage of associativity and needs to be gone into carefully. When we multiply by 20, we do it in two steps. We treat 20 as 2×10 and first multiply the number by 2, and then by 10. Teachers sometimes use language incorrectly here by saying,"Multiply by 2 and add zero to the answer". It is better to say "Place a zero next to the answer".

ACTIVITY *THIRTEEN* | Using multiplication facts to get new facts

Materials required: Place value Kit.

Encourage children to do mental arithmetic and find efficient ways of doing multiplication.

As children become conversant with the three laws, they will be able to use them in simplifying multiplications. For example:

To do $4 \times 8 \times 25$, they may first multiply 4×25 to get 100, and then multiply by 8 to get 800.

To do 7×35, they may first do $7 \times 30 = 210$ and then $7 \times 5 = 35$, and then add 210 and 35 to get 245.

They may use a 'halving and doubling' technique. To do 16×4, they may multiply 8 (half of 16) with 8 (double of 4).

They may use 'rounding and subtraction'. To do 28×5, they may do: $(30 - 2) \times 5 = 30 \times 5 - 2 \times 5 = 150 - 10 = 140$.

ACTIVITY ***FOURTEEN*** | Multiplication of a double digit number by a double digit number

Children face a lot of difficulty in understanding the procedure of double digit multiplication and many errors happen in this area. A chief cause of this problem is focusing on procedures mechanically and not paying sufficient attention to the logic of the procedure and the concept.

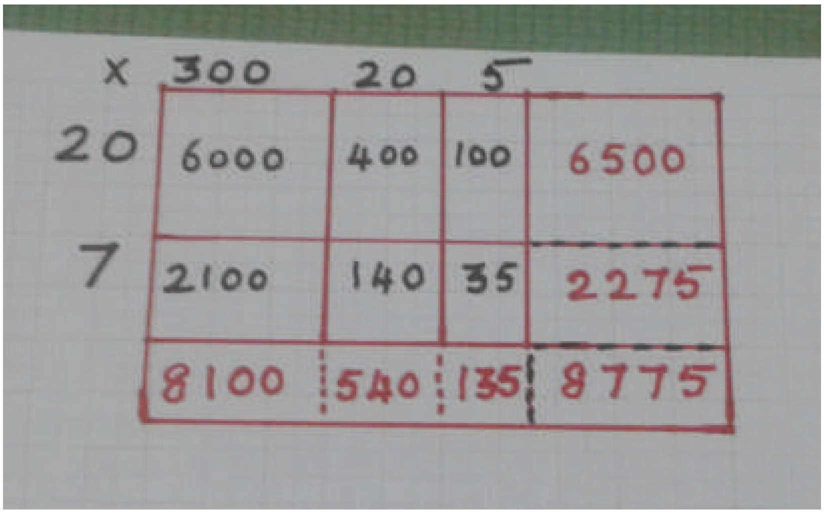
Multiplication by double digit involves the distributive law, place value and the usage of zero as a place holder. When we multiply 24×32, one needs to show it initially by writing it in full expanded form as $24 \times 30 + 24 \times 2$.

24×30 in turn can be seen as $24 \times 3 \times 10$ which is 72 tens (720), and $24 \times 2 = 48$. Repeatedly one needs to draw the child's attention to the place value of the digits with which we are multiplying.

While multiplying with the number in the tens place, one must start by placing a zero in the units place. Leaving it blank, or using some other symbol like a star or a cross, does not make sense, nor does it aid in understanding what is happening.

Multiplication of a three digit number by a two digit number can be shown through expanded form and partial products.

Ex.325×27:



Extension: Similarly, when we multiply a number by a three digit number, we must point out that multiplying by a digit in the hundreds place will result in zeroes in the units and tens places in the product.

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