# DAY 4

## Pedagogy of Mathematics

*Guidelines for the trainers: The following section contains details about skill sets and teaching aids required to conduct the training session along with references for further reading.*

| **Required trainer skill set:*** Understanding of Maths curriculum and Textbook
 | **Expected Outcome:** After going through this unit, Participants will be able to* Understand aims of teaching and learning mathematics education.
* Identify the indicators of effective mathematics education.
* Create a conducive learning environment in the school for Mathematization.
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| **Day** | **Sub-topic** | **Concepts** | **Pedagogy** | **Time** |
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| Day 4 | *Competency Developed:* Knowledge of pedagogy, Applying Multiple Teaching Style and instructional strategies to engage student in learning process |
| Pedagogy of Mathematics |  | Activity 1: Why Mathematics?  | 90 Mins |
| Break | 10 Mins |
| Activity 2: Understanding the usages of different materials and tools which can be used to solve mathematical problems.  | 90 Mins |
| Lunch Break | 60 Mins |
| Activity 3: Structure discussion around Maths education | 70 Mins |
| Break | 10 Mins |
| Activity 4: Structured reflection | 60 Mins |

| **Activity 1:** Why Mathematics? **Objective:** Identify the indicators of effective mathematics education.**Output:** Participants will be able to understand key indicators for learning mathematic**s.** **Resource Required:** Computer, Notebook, Pens, Chatpapers, Sketch Pens**Procedures:** Step 1: Trainer will ask the following Questions to the entire class for initiating the discussion. Questions are * Which are the areas and everyday works where mathematics holds a key position.
* Why do you think that children should learn Mathematics?

Step 2: Trainer will ask participants to discuss in their respective small groups, then write down their answers on chart papers. Step 3: Trainer will explain the Lecturer notes and conclude the session. While discussing the concepts, Participation from learners will be welcome. It helps in bringing new relevant examples. **Lecture Notes:** In general, mathematics refers to the application of concepts, procedures and methods developed in mathematics. Developing the ability of mathematization which is regarded as constituting the higher aims of mathematics, includes developing such abilities as problem solving, use of heuristics, estimation and approximation, optimisation, use of patterns, visualization, representation, reasoning and proof, making connections, mathematical communication including developing aesthetic feeling. **Concluding Discussion:** * **Problem Solving:** The problem solving skills include skills of observation, experimentation, estimation, reasoning and verification. Example: How can we measure the length of a classroom? Students may utilise their experience of measuring a table and can use fingers, foot, hand, stick, rope, measuring tape etc. After some trials students may use measuring tape when they are asked to measure a long space. When children learn a variety of approaches their toolkit gets richer experiences. While solving these mathematical problems in the classroom, the student may just understand it as a task to be done. Here it is important for the teacher and supervisors supporting teaching to relate these mathematical problems and relate it to daily life. This enables them to understand that they can apply their mathematical learning in problem solving.
* **Use of Heuristics:** It is generally believed that mathematics is considered to be ‘exact’ where one uses ‘the appropriate formula’. But one can use alternative processes and interactive methods to solve a problem. This encourages the learner to try different hunches for solving the problems. Example: There are 10 students in the classroom. Students will shake hands with each other. How many total handshakes? There are multiple ways that we can solve this puzzle. a. calculating one by one 2. Taking a relative a small number and trialing it. 3. By using the theory of permutation. Therefore in a mathematical classroom, it is important for teachers to provide exposure to students so that students understand the problem and learn to solve it by applying different methodology.
* **Estimation and approximation:** In many cases students use this skill to employ these approximations in solving more complex problems. School mathematics, therefore, can play a significant role in developing and refining such useful skills. Example: In a projectile motion (throwing a ball), concepts like angle, speed, distance travelled, Direction, Height etc were taught to students. Students in the classroom are merely responsible to answer the questions rather than asking questions. The teacher not only should focus on supporting them learn estimation and appropriation but also need to provide its application in life experiences as well.
* **Optimization:** means utilization of available conditions and resources to the fullest extent. While teaching loan and interest in specific grade, the student may just learn how to solve compound interest and simple interest problem but it is upto to the t Example: Take an example of Economic reasoning/Finance Planning. Mr. Iqbal annual income is 3.5 lakhs Afgani. He wants to set up an automobile repair shop worth 15 lakhs afgani for his daughter. He would like to take a bank loan, how many years can he take out a loan to settle it? (It depends on Age of the borrower, Savings Mr. Iqbal have, Installment Calculation, compound interest etc.)
* **Reasoning and Proof:** The process of reasoning and proof is important in mathematics. The aim should be to encourage proof as a systematic way of argumentation, evaluate arguments, make and investigate and understand that there are various methods of reasoning. Example: What next?

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89* **Use of patterns:** Study of patterns requires students to recognize, describe, and generalize patterns (events, shapes, designs, sets of numbers) to arrive at rules and formulae.

Example: Doors/Window Design |
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| **Activity 2:** Understanding the usages of different materials and tools which can be used to solve mathematical problems. **Objective:** To provide participants with opportunities to understand mathematical concepts through material design.**Output:** To develop a prototype to understand mathematical concepts. **Resource Required:** A4 Sheet Papers, Scissors, Whiteboard, PPT & Computers**Procedures:** * Step 1: Trainer reference video:<https://www.youtube.com/watch?v=8y9i8ag2WTk>
	+ (Trainer will Practice before facilitation)
* Step 2: Instruction to participants: Trainer will ask the participants in the small groups “How they can pass through a A4 Sheet Paper”
* Step 3: After distributing A4 Sheets to all groups, It is expected that a small group will discuss within group members and come up with a solution.
* Step 4: There will be three kinds of group engagements. a. Some groups will solve the puzzle fast b. Some of them will have different guesses c. Few of them will not be able to understand the problem.
* Step 5: Here are the trigger Questions that might help groups to solve the challenge. How we can cut the to expand the area of the paper.
* Step 6: Trainer will demonstrate the exercise.

**Concluding Discussion:** * Manipulative materials are objects or things that the learner is able to feel, touch, handle, and move. They may be real objects which have social application in our everyday life. In the above exercise we use A4 Sheet Paper. There are many such objects available which can be used to demonstrate mathematical concepts. Example: Matchsticks, broken Tiles, clothes, straws, weighing scale etc. Here it is important to understand that learning mathematics is not restricted to solving textbook problems and secondly learning material to construct knowledge experiences in a mathematical classroom is not always expensive.
* When a teacher tries to teach the concept of Area and Perimeter. A4 Sheet paper is given (**210 x 297 Millimeters**). So its perimeter will be 1014 millimeters. The size of paper is so small that it is not possible for a human to pass through it. So we need to cut the paper to expand its perimeter. Here, a simple A4 paper sheet not just provides an opportunity to solve the problem but also provides opportunity by engaging students in handson activity. After solving the problem, here it is important for the teacher to understand what kind of trigger question can make this exercise more meaningful.For example: Does cutting the paper help in increasing the area?
* For practice, the Trainer may ask the participant to Try other manipulative materials at Home: One material that can be used to teach a mathematical concept. Link to Hand-on Maths guide: <https://www.arvindguptatoys.com/arvindgupta/e-hands-on-maths.pdf>
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| **Activity 3:** Structure discussion around maths education**Objective:** To make participants understand the various methods of teaching mathematics. **Output:** Participants can apply these methods to appreciate mathematics teaching. **Resource Required:** computer, Projector, Notebooks, Pens**Procedures:** Step 1: Trainer will ask the participants to go through the reading. (Teaching Multiplication Visual Approach)Step 2: After the Reading, the Trainer will ask participants what they have understood from it? Trainer will write down the responses on the whiteboard. Few Additional Questions that might help the trainer to get more answers. * Why do you think that this method is interesting?
* Do you think that learners will understand multiplication better through this process?
* Do you think that we may come out of rote learning by using such techniques?

Step 3: After completing the above step (which takes approx 10 Mins), Trainer will explain as follows. * Let's not talk about teachers and her teaching. Discuss about the techniques itself to understand multiplication.
* It is tempting to appreciate such techniques. It seems that the entire class is engaged. In our everyday life, We observe different kinds of mathematical patterns that surprise us. Selection of those patterns can be part of the teaching learning process that requires critical evaluation and thinking.
* In our concluding session, we will see a few methods of doing mathematics. It will help us to evaluate techniques that can be used in classroom practices.

**Concluding Discussion:**  The following are the few methods that We used to make the teaching-learning process of Mathematics effective. * Inductive-Deductive Method

Inductive method is to move from specific examples to generalization and deductive method is to move from generalization to specific examples. Things like Formulas, theorems, examples, results are derived, proved and used. Let's take a few examples. Example 1: Prove that 5n+1 is an odd number. Where n is a natural number. Let's consider n=1, 2, 3, 4 , 5. If we put the value of n in the above mentioned equation, We get values 7, 11, 16, 21, 26 respectively. It is clear that 5n+1 is not an odd number. This is an inductive reasoning process. We tested the different conditions, found the irregularities and then we generalised that the given unknown is not an odd number. Deductive reasoning Example 2: All numbers ending with either 0 or 5, are divisible by 5. So find out which of the following numbers is divisible by 5? (In Deductive reasoning, Generalisations are given/proved. We need to identify cases from the generalisation statement. Option A: 1055 Option B: 100 Option C: 75 Option D: All of the above* Analytic and Synthetic Method

Analytic is breaking down and moving from unknown to known and Synthetic is putting together known bits of information and moving from known to unknown.) These methods are basically used in proving the results and solving sums. In textbooks mostly synthetic methods are used, to prove something unknown. Trainer may ask them to check a few math textbooks to verify this. Example: *if a/b*=*d/c then* Prove that *d(a-2ab) = b(c-2ad)*

| Analytical Method | Synthetic Method |
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| *a/b*=*d/c*∴*b/a*- *2a =d/c*- *2a* ∴*d(a-2ab) = b(c-2ad)* | *d(a-2ab) = b(c-2ad)*⇔*(a* − 2*ab)/b=(c* − 2*ad)/d* ⇔*b/a*- *2a =d/c*- *2a* ⇔*b/a*=*d/c* |

* Problem Solving Method

Presenting the knowledge to be learnt in the form of a problem. It begins with a problematic situation and consists of continuous meaningful well-integrated activity. Example 1: Suppose there are 50 people in the classroom, each of them will shake hands with each other. How many total handshakes will be there? Note for Facilitator: It is an example of Arithmetic Progression. 1+2+3+......+N=N\*(N+1)/2 [Total handshakes will be 1+2+3+.....49 = 49\*(49+1)/2 = 1225Laboratory Method: Learning by doing is one of the most effective ways to learn mathematical concepts. It helps learners to proceed from concrete to abstract. * Let's Take an Example: Suppose School is going to buy carpet for classrooms? How much carpet will be sufficient enough? So Find the Area of the classroom. A simple meter Tape is sufficient to demonstrate this exercise.
* Use of ICT is also helpful here. Try <https://phet.colorado.edu/sims/html/area-model-introduction/latest/area-model-introduction_en.html>
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| **Activity 4:** Structured reflection**Objective:** To connect today's learning into their Job responsibility. **Output:** To discuss, reflect and submit the answers in diarie**s.** **Resource** Required: Notebook and pen**Procedures:** * Trainer will ask them to list out two ideas/ skills/ strategies that were most useful for them in today’s training, and one idea/ skill/ strategy that they need clarification on. + Multiple Trainers will answer questions in small groups for the supervisors, ensuring that everyone gets time to share in their groups.
* All the structured reflection sessions can also be used for further contextualization of the learning. Participants can reflect about the skills and strategies they learned, challenges they see with the existing supervision system, and how best they can be used to improve on them.

**Trainer Notes:** The participants will end the day with the structured reflection session. To conclude the structured reflection session, participants and trainers will engage in an interactive diary session throughout the 10 days of training. Interactive dairy is a strategy to include the supervisors’ voice in the training. In an interactive diary, the trainees get to give live feedback on the design of the training that they are undertaking by engaging in a dialogue with the trainer. Other than this, there are two reasons to include interactive diaries in this training. First, it will include supervisors’ voice in the training process. Second, it builds a transferable skill in the supervisors that they can use when they are working in the field with their supervisors. The Trainers will see trainees as valuable contributors in their development as supervisors, and for trainees get to actually comment on their training to make it their own. **Activity:** The diaries will be distributed to the supervisors along with the prompts for feedback on the training, such as,* How did ancient Mathematics contribute to modern day life?
* How does Mathematics affect our daily lives? (While cooking: by using ingredient we use promotional reasoning, All kind of Transaction with Bank, at Market to buy groceries, Fruits etc. )
* Why are women underrepresented in Mathematics fields ?
* Is there some new way in which the teacher can present a concept in order to make it more meaningful and more interesting?
* What activities, demonstrations, teaching aids, etc. would enrich the classroom presentation? (Take any textbook example)
* How can you support Maths teachers as part of your responsibility? (Your own preparation, Strategies with reference to National Goal)

Supervisors will write their notes for the Trainers in the diaries and then these will be collected. The trainers will respond to the diary entries by writing back, thus beginning a dialogue between the Trainers and the trainee supervisors.  |
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\*\*\*\*\*\*Day End\*\*\*